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MICROPARTICLE DYNAMICS IN A NONLINEAR ELECTROMAGNETIC PAUL TRAP

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Axial motion stability was investigated for an ion confined within a nonlinear Paul trap, which undergoes interaction with a resonant laser field. We have studied and illustrated the phase portraits, and the competition between the radiofrequency field, laser field, micromotion and anharmonicities in the transition from chaos to order.

Key words: Paul trap, laser cooling, Lamb-Dicke regime, secular approximation, phase portraits, multipolar anharmonicities, micromotion, attractor, chaos.

1. INTRODUCTION

Ion traps, in which stored ions are generated and confined for long periods of time, allow performing important and conceptually new experiments in the fields of quantum electrodynamics and quantum optics [1, 2], ultrahigh precision spectroscopy [3], quantum metrology, new time and frequency standards [4, 5], as well as the realization of quantum computers [6, 7]. The interaction between a single trapped ion and a laser field [8] has become a problem of considerable interest in the recent years. A particular interest rises the situation in which the laser field is quasiresonant with a metastable transition of the trapped ion [9]. From an experimental point of view, forbidden dipole transitions have been used to cool an In^+ or Hg^+ ion [10, 11] down to very low temperatures through sideband laser cooling and to measure the ion temperature through the quantum jumps technique. Theoretically, dissipation in the system is negligible which leads to a number of interesting phenomena [12].

Ion traps feature unique characteristics such as long interaction times, absence of collisions with the walls of the containing vessel, the possibility to trap a single ion and to localize particles to space regions which are less than an optical wavelength. Thus a trapped ion not only represents a well defined light source for studying the quantum effects of radiation [10], but the strongly nonlinear character of the associated dynamics turns these traps into ideal candidates for tests on integrability and chaotic behaviour in classical and quantum mechanics [4, 12]. Dynamical aspects such as crystal – cloud phase transitions related to the transition from the regular motion regime towards a chaotic one, yield to an unprecented interest [11, 12]. Crystallization phenomena of trapped and cooled ions have increasing importance for the realization of new and miniaturized atomic frequency standards, with performances better than the present ones. Thus, laser cooled ions confined in the dynamic potential of a Paul trap present all the standard characteristics of chaos. The trap potential may be treated, to a good approximation, as a quantum mechanical harmonic oscillator.

Most of the theoretical analysis of the interaction between laser fields and ions without dissipation, is based on two approximations which have been used: (1) the so-called Lamb-Dicke regime and (2) the secular approximation. In the Lamb-Dicke limit, ion motion is restricted within a region which is small compared to the laser wavelength, a fact which introduces significant simplifications in the problem of interest [4]. The secular approximation is related to the fact that for ions confined within a Paul (radiofrequency – RF) trap, the time dependent trapping potential can be replaced with a harmonic potential. This fact reduces to averaging the ion motion over short time intervals, thus neglecting the ion quick oscillations (micromotion) with respect to its slow motion (secular motion). In fact, in the Lamb-Dicke limit and for laser frequencies not far away from the atomic resonance, the last assumption is very well satisfied in the context of laser cooling. In particular, ion micromotion within a Paul trap can become important and can lead to other phenomena not considered so far. It is obvious that a qualitative analysis and a complete description of this problem would be desirable, taking into account ion micromotion within the trap. The aim of this paper is to analyze axial stability of a confined ion, in presence of damping and laser cooling, considering the micromotion effects.

2. THE EQUATION OF MOTION FOR AN ION

This article deals with the stability of an ion confined in a multipolar electromagnetic Paul trap, nonlinear, with higher order (4th and 6th order) anharmonicities. This type of trap represents the basis of new atomic time-frequency standards with superior performances compared to state of the art ones, a characteristic which would explain the intense attention paid to the realization of new trap geometries which would enhance the number of trapped particles (namely the signal-noise ratio) and minimize the second order Doppler shift. For reasons of simplicity, the effects owed to micromotion have been neglected in the research performed so far. Also, in order to further simplify the problem mostly ideal quadrupolar trap potentials have been considered, corresponding to the first two lower order terms in the series expansion of the trap potential. In reality, quadrupolar traps are far from being ideal and geometric imperfections which are inherent in the realization of such traps (in spite of the fact that tolerances are not larger than tens of millimeters) make the trap potentials to differ from the ideal ones. As a consequence, octopolar and multipolar terms appear which in certain situations can result in a pronounced instability and in the occurrence of chaos in the ion dynamics. Moreover micromotion effects can lead to large instabilities in the particle dynamics, our analysis considering these effects. We also tried to emphasize the appearance of attractors in the particle dynamics. An attractor presents a certain stability: any trajectory starting from its vicinity can move away, but eventually returns in order to tend to it asymptotically. The effects owing to air friction (for traps operating in air) or laser damping [13] have been also considered by introducing a term which describes damping in the equation of motion.

We have investigated the dynamics of a particle (ion) confined within a nonlinear quadrupolar trap, with higher order anharmonicities resulting from an multipolar (octopolar) potential. The trapped ion was assimilated to a time periodic differential dynamical system. In order to study the competition between micromotion and multipolar anharmonicities, the following equation of axial motion was considered for a charged particle of mass M and charge Q, in a quadrupolar Paul trap with anharmonicity $\varepsilon(z^6/6+z^4/4)$, resulting from a multipolar potential, in presence of damping and of a perturbative electromagnetic field:

$$\frac{d^2u}{d\tau^2}+\gamma\frac{du}{d\tau}+[a-2q\cos(2\tau)]u+\lambda_1 u^5+\lambda_2 u^3+\alpha\sin u=f(t), \tag{1}$$

where the adimensional a and q parameters are:

$$a=-16QU_0/M\Omega^2\left(r_0^2+2z_0^2\right) \tag{2a}$$

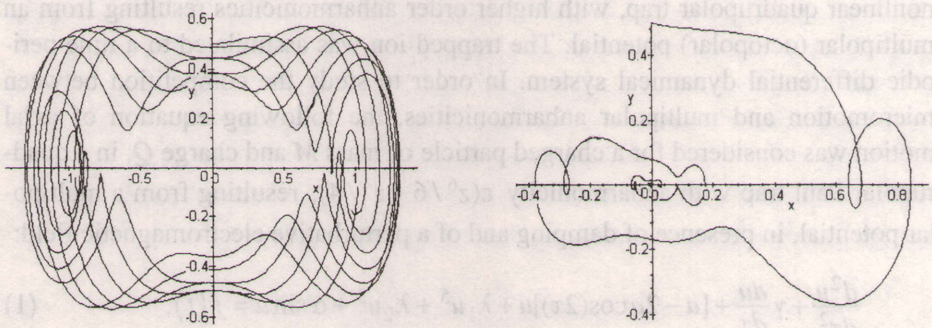
$$q=8QV_0/M\Omega^2\left(r_0^2+2z_0^2\right), \tag{2b}$$

and $\tau=\Omega t/2$, $u=kz$, $\alpha=2k^2\Omega_0\cos\theta/M\Omega^2$, where r_0 and z_0 are the semiaxes of the quadrupolar Paul trap. The term Ω stands for the micromotion frequency, Ω_0 is the Rabi frequency of the ion-laser interaction, while $\cos\theta$ is the expected value of the spin projection operator z for the system with two energy levels considered with respect to a Bloch coherent state. The terms in z^5 and z^3 represent the higher order nonlinear contributions of the multipolar (octopolar) potential. The $\alpha\sin u$ term in the equation of motion (1) describes the interaction of the differential dynamical system with the laser radiation, while $f(t)=\delta\sin t$ stands for the kicked term.

3. PHASE PORTRAITS

In order to illustrate ion dynamics in the nonlinear quadrupolar trap described by the law of motion (1) we considered, we have studied the trajectories in the phase space (phase portraits) for different control parameters. As shown in the following, these emphasize the existence of strange attractors and fractal basin boundaries for the ion dynamics.

Fig. 1 a and b shows the phase portraits of the parametric oscillator and of the damped parametric oscillator. In case of the parametric oscillator, the existence of two attractors is obvious. Friction in air (in case of a trap operating under standard temperature and pressure conditions) or due to interaction with the laser radiation results in a damping of the ion motion, as seen in case (b). Thus, ion motion is restricted to the trap centre where the amplitude of the trapping field has a vanishing point. Hence, ion stability is strongly increased, while the trapping time can theoretically have an infinite value.

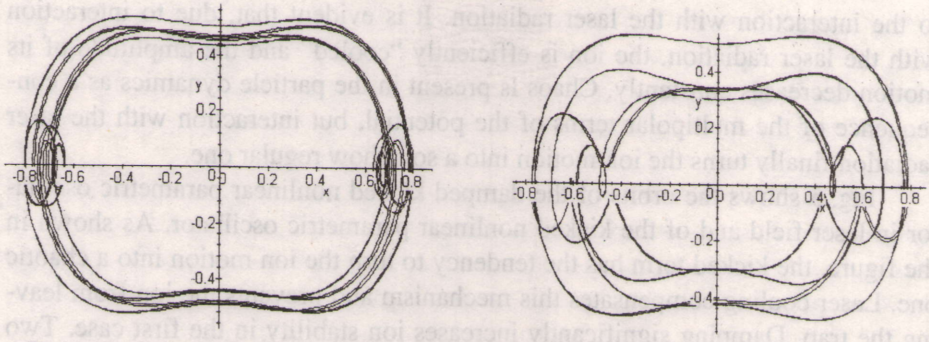


Parametric oscillator Damped parametric oscillator

Fig. 1. – Phase space orbits for the parametric oscillator ($\gamma = 0$, $\lambda_1 = \lambda_2 = 0$, $\alpha = 0$, $\delta = 0$) and damped parametric oscillator ($\gamma = 0.2$, $\lambda_1 = \lambda_2 = 0$, $\alpha = 0$, $\delta = 0$) with initial conditions $(z,\dot{z}) = (0,0.5)$.

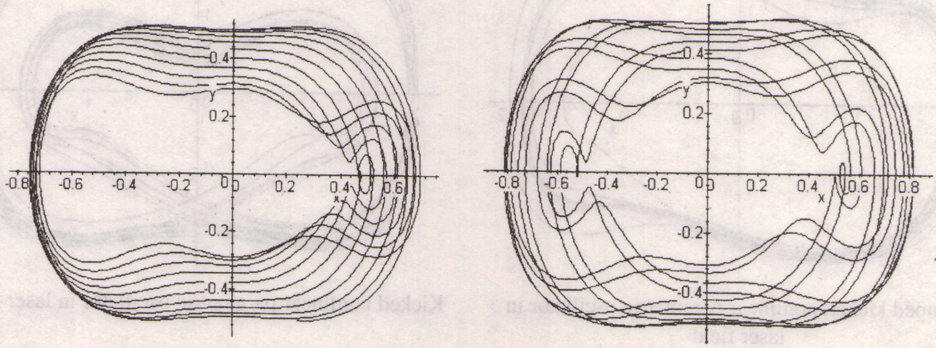
In Fig. 2, the nonlinear parametric oscillator is presented. It can be observed that the nonlinear terms in z^3 and z^5 worsen the particle motion stability, which leads to the appearance of chaos. The existence of two attractors for the ion dynamics is obvious in both cases, as well as trajectories compactization due to the damping term. It is clear that laser cooling and damping decreases the particle energy and the amplitude of the motion, while enhancing ion trapping time. Laser cooling also reduces chaotic effects in the ion motion, induced by the nonlinear terms. The phase potrait in case $\lambda_1 = 0.2$ and $\lambda_2 = 0$ is very similar to case 1.

Figures 3 and 4 present the orbits of the nonlinear parametric oscillator in laser field, for different control parameters. The existence of attractors for the ion dynamics is again obvious, as well as the break of symmetry in case when only one of the nonlinear terms (z^3 or z^5) is present. The trajectories are compact due



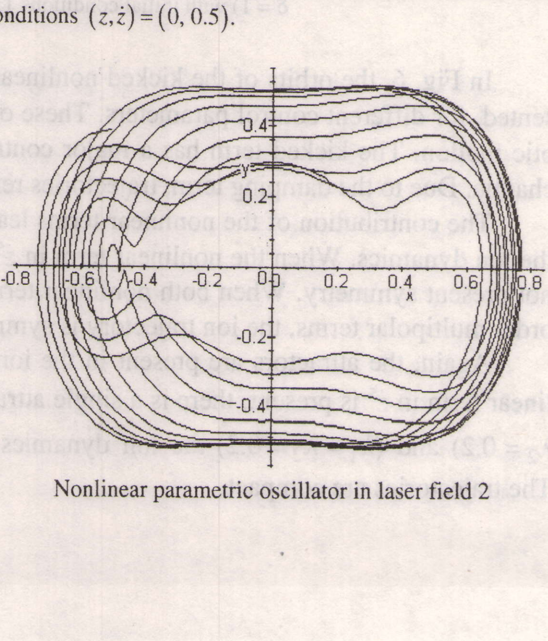
Damped nonlinear parametric oscillator   Damped nonlinear parametric oscillator 2

Fig. 2. – Phase space orbits for the nonlinear parametric oscillator in cases 1 ($\gamma = 0.2$, $\lambda_1 = 0$, $\lambda_2 = 0.2$, $\alpha = 0$, $\delta = 0$) and 2 ($\gamma = 0.2$, $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $\alpha = 0$, $\delta = 0$) with initial conditions $(z, \dot{z}) = (0, 0.5)$.



Nonlinear parametric oscillator in laser field 1   Nonlinear parametric oscillator in laser field 2

Fig. 3. – Phase space orbits for the nonlinear parametric oscillator for control parameters ($\gamma = 0.2$, $\lambda_1 = 0$, $\lambda_2 = 0.2$, $\alpha = 0$, $\delta = 0$) in case 1 and ($\gamma = 0.2$, $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $\alpha = 0$, $\delta = 0$) in case 2, with initial conditions $(z, \dot{z}) = (0, 0.5)$.

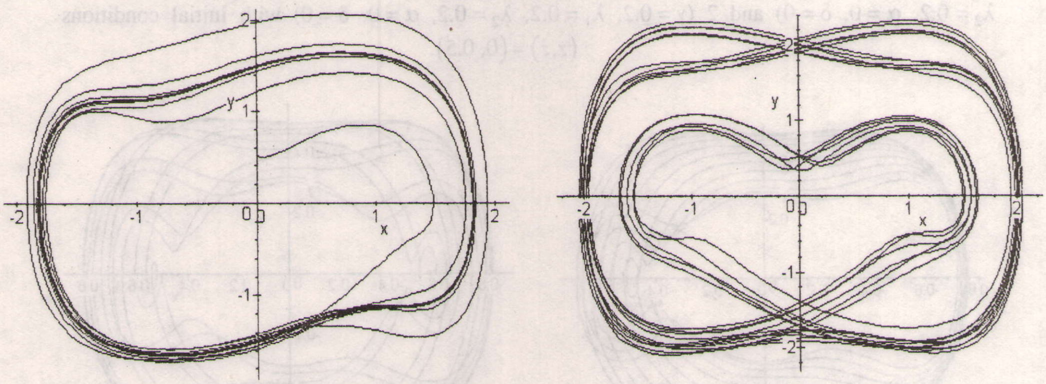


Nonlinear parametric oscillator in laser field 2

Fig. 4. – Phase space orbit of the nonlinear parametric oscillator in laser field for control parameters ($\gamma = 0$, $\lambda_1 = \lambda_2 = 0.2$, $\alpha = 0.2$, $\delta = 0$) with initial conditions $(z, \dot{z}) = (0, 0.5)$.

to the interaction with the laser radiation. It is evident that, due to interaction with the laser radiation, the ion is efficiently "cooled" and the amplitude of its motion decreases constantly. Chaos is present in the particle dynamics as a consequence of the multipolar terms of the potential, but interaction with the laser radiation finally turns the ion motion into a somehow regular one.

Fig. 5 shows the orbits of the damped kicked nonlinear parametric oscillator in laser field and of the kicked nonlinear parametric oscillator. As shown in the figure, the kicked term has the tendency to turn the ion motion into a chaotic one. Laser cooling compensates this mechanism and prevents the ion from leaving the trap. Damping significantly increases ion stability in the first case. Two attractors are clearly present in the second case.



Damped kicked nonlinear parametric oscillator in laser field

Kicked nonlinear parametric oscillator in laser field

Fig. 5. – Orbits of the damped kicked nonlinear parametric oscillator in laser field ($\gamma = 0.2$, $\lambda_1 = 0.2$, $\lambda_2 = 0$, $\alpha = 0.2$, $\delta = 1$) and of the kicked nonlinear parametric oscillator in laser field ($\gamma = 0$, $\lambda_1 = 0.2$, $\lambda_2 = 0$, $\alpha = 0.2$, $\delta = 1$) with initial conditions $(z, \dot{z}) = (0, 0.5)$.

In Fig. 6, the orbits of the kicked nonlinear parametric oscillator are represented, for different control parameters. These orbits are characteristic for a chaotic motion. The kicked term has a major contribution in rendering the motion chaotic. Due to the damping term, trajectories remain compact.

The contribution of the nonlinear terms leads to the appearance of chaos in the ion dynamics. When the nonlinear term in z^5 is dominant, the trajectory does not present symmetry. When both nonlinear terms are present, due to the higher order multipolar terms, the ion trajectory is symmetrical.

Again, the attractors are present in the ion dynamics. When only the nonlinear term in z^5 is present, there is a single attractor, while for the cases ($\lambda_1 = 0$, $\lambda_2 = 0.2$) and ($\lambda_1 = \lambda_2 = 0.2$) the ion dynamics clearly presents two attractors. The trajectories are compact.

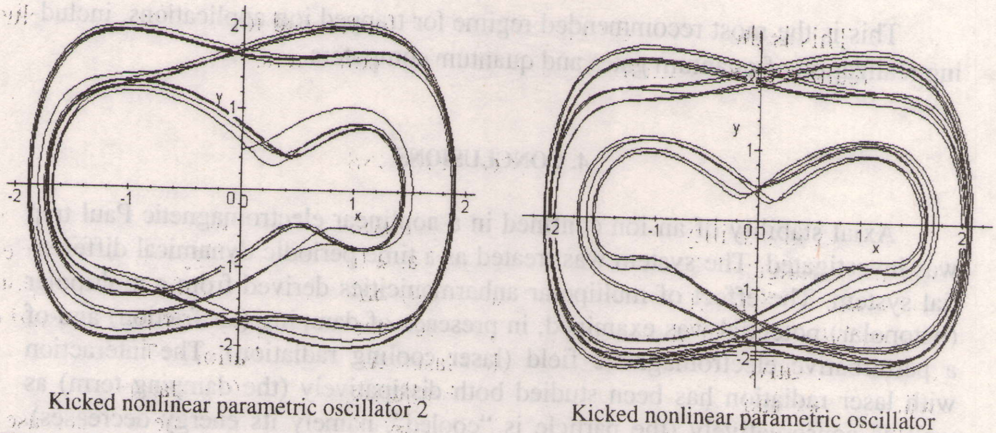


Fig. 6. – Orbits in the phase space for the damped nonlinear parametric oscillator for control parameters 1 ($\gamma = 0.2$, $\lambda_1 = 0.2$, $\lambda_2 = 0$, $\alpha = 0$, $\delta = 0.2$) and 2 ($\gamma = 0.2$, $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $\alpha = 0.2$, $\delta = 0.2$) with initial conditions $(z, \dot{z}) = (0, 0.5)$.

Finally, Fig. 7 shows the orbit for the damped nonlinear parametric oscillator in laser field. As it is evident, both damping and laser radiation efficiently "cool" the ion and severely decrease its energy. Hence, the ion orbit amplitude falls drastically and rapidly. The ion is finally confined in a very small region, practically in the trap centre.

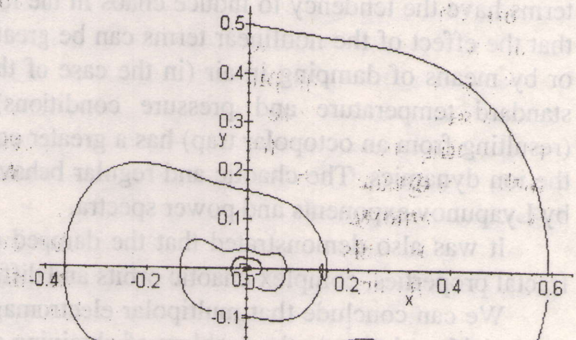


Fig. 7. – Phase space orbit for the damped nonlinear parametric oscillator in laser field for control parameters ($\gamma = 0.2$, $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $\alpha = 0.2$, $\delta = 0$) with initial conditions $(z, \dot{z}) = (0, 0.5)$.

We can ascertain that this situation corresponds to the Lamb-Dicke regime, when the ion is trapped within a space region small compared to the wavelength of the laser radiation. This is the regime which is most often used, where side-band laser cooling can be applied. This regime is fundamental for the realization of new atomic time-frequency standards, and has a wide range of applications for quantum optics and electrodynamics. High-precision spectroscopy experiments are also performed in this regime.

This is the most recommended regime for trapped ion applications, including realization of quantum gates and quantum computers.

4. CONCLUSIONS

Axial stability of an ion confined in a nonlinear electromagnetic Paul trap was investigated. The system was treated as a time periodic dynamical differential system. The effect of multipolar anharmonicities derived from a multipolar (octopolar) potential was examined, in presence of damping (air friction) and of a perturbative electromagnetic field (laser cooling radiation). The interaction with laser radiation has been studied both dissipatively (the damping term) as well as conservatively (the particle is "cooled", namely its energy decreases). Micromotion effects have been also considered.

Trajectories in the phase space have been investigated and represented, both numerically and graphically, regular and chaotic orbits have been identified as well as the competition between the radiofrequency field, laser field, micromotion and anharmonicities in the transition from chaos to order. The appearance and existence of attractors for the trapped ion dynamics has been emphasized. It was also pointed out that both damping (friction) and the interaction with the laser radiation result in compact trajectories in space. We demonstrated that, for sufficiently high control parameters λ, the higher order nonlinear terms have the tendency to induce chaos in the ion dynamics. It was also shown that the effect of the nonlinear terms can be greatly diminished by laser cooling or by means of damping in air (in the case of the traps operating in air, under standard temperature and pressure conditions). The nonlinear term in z^3 (resulting from an octopolar trap) has a greater contribution in inducing chaos in the ion dynamics. The chaotic and regular behaviour of the ion is characterized by Lyapunov exponents and power spectra.

It was also demonstrated that the damped parametrical oscillator presents fractal properties, complex chaotic orbits and different attractors.

We can conclude that multipolar electromagnetic traps (hexa or octopolar) are a viable solution to the problem of obtaining a higher signal-noise ratio and a reduced second order Doppler shift, but care should be taken in order to minimize the contributions of the nonlinear terms as well as the micromotion contribution. A perfect candidate which meets these requirements is represented by the linear geometry Paul traps. Their advantage is the possibility to minimize, by construction, the contributions of the higher order multipolar terms in the series expansion of the trap electric potential. Thus, multipolar electromagnetic traps can be achieved, which will enable confinement of a larger number of ions (particles), while in the same time diminishing the contributions of the perturbative factors due to the inherent mechanical imperfections when realizing quadrupolar traps with geometries other than linear.

Multipolar traps are dedicated to the development of the new trapped ion atomic frequency standards, with enhanced performances compared to state of the art ones, as well as for performing fundamental quantum optics and electrodynamics experiments, with extremely high quality factor atomic resonances.

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REFERENCES

1. I. Siemers, R. Schubert, R. Blatt, W. Neuhauser, and P. E. Toschek, Europhys. Lett. **18**, 129 (1992).
2. Ch. Roos, Th. Zeiger, H. Rohde, H. C. Nägerl, J. Eschner, D. Leibfried, F. Schmidt-Kaler and R. Blatt, Phys. Rev. Lett. **83** (**23**), (1999).
3. J. C. Bergquist, F. Diedrich, W. M. Itano and D. J. Wineland, in *Laser Spectroscopy IX*, edited by M. S. Feld, J. E. Thomas and A. Mooradian (Academic, San Diego, 1989), p. 274.
4. See J. Mod. Opt. **39**, 192 (1992), special issue on *The Physics of Trapped ions*, edited by R. Blatt, P. Gill and R. C. Thompson.
5. L. S. Brown, Phys. Rev. Lett **66**, 527 (1991); A. Lindner, Z. Phys. **D33**, 43 (1995); S. R. Jefferts, C. Monroe, E. W. Bell and D. J. Wineland, Phys. Rev. **A51**, 3112 (1995).
6. J. F. Poyatos, J. I. Cirac and P. Zoller, Phys. Rev. Lett. **78**, 390 (1997) and Phys. Rev. Lett. **81**, 1322 (1998).
7. H. C. Nägerl, W. Bechter, J. Eschner, F. Schmidt-Kaler, R. Blatt, Appl. Phys. **B66**, 603 (1998).
8. H. C. Nägerl, D. Leibfried, H. Rohde, G. Thalhammer, J. Eschner, F. Schmidt-Kaler and R. Blatt, Phys. Rev. **A60**, 145 (1999).
9. J. I. Cirac, R. Blatt, A. S. Parkins and P. Zoller, Phys. Rev. **A48**, 2169 (1993).
10. E. Peik, G. Hollemann and H. Walther, Phys. Rev. **A49**, 402 (1994).
11. M. G. Raizen, J. M. Gilligan, J. C. Bergquist, W. M. Itano and D. J. Wineland, Phys. Rev. **A45**, 6493 (1992).
12. J. Hoffnagle and R. G. Brewer, Phys. Rev. **A50**, 4157 (1994); R. Chacón and J. I. Cirac, Phys. Rev. **A51**, 4900 (1995).
13. R. G. DeVoe, J. Hoffnagle and R. G. Brewer, Phys. Rev. **A39**, 4362 (1989).